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Towards a New Concept of Closed System: from the Oscillating Universe to the EM-Drive

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Abstract**—***The EM-Drive, as long as it is considered a closed system, explicitly violates the conservation of momentum and Newton's well-known third law: however, it would appear, according to several tests to date, that the device may concretely deliver a certain thrust without a detectable exhaust. The question is: can the EM-Drive be actually regarded as a closed system? We have elsewhere tried to provide a qualitative answer by resorting to a theory based, amongst other hypotheses, upon the existence of a further spatial (hidden) dimension. In this paper, the whole revised theory is step-by-step expounded, avoiding, for the sake of brevity, some aspects that, notwithstanding their undeniable relevance, do not concretely contribute to the achievement of our main goal. We consider a Universe belonging to the so-called oscillatory class. Firstly, we formally show that, as it is well known, a simple-harmonically oscillating Universe is fully compatible with General Relativity. Then, we carry out an alternative deduction of the mass-energy equivalence formula as well as of the Friedmann–Lemaître equations. Finally, by resorting to an opportune writing of the conservation of energy (carried out by taking into account the alleged extra spatial dimension), we implicitly obtain a new definition of closed system, so providing an answer to the question previously posed.*

Keywords**—*****Closed System, Oscillating Universe, Extra Dimension, Friedmann–Lemaître Equations, EM-Drive, Mass-Energy Equivalence, Relativistic Energy, Resonant Cavity Thruster, Reflectors Temperature.***

I. THE OSCILLATING UNIVERSE

1. Uniform Cosmological Models: Oscillatory Class

For a uniform Universe, with the usual hypotheses of homogeneity and isotropy, we can write the first Friedmann – Lemaître Equation [1] as follows:

$$\left(\frac{dR}{dt}\right)^2=\frac{1}{3}(8\pi G\rho+\lambda c^2)R^2-kc^2 \tag{1}$$

R represents the scale factor, G the gravitational constant, ρ the density, λ the so-called cosmological constant, k the curvature parameter, whose value depends on the hypothesized geometry, and c the speed of light.

As well known, if we denote with E energy, with T the thermodynamic temperature, with S the entropy, with p the pressure, and with V the volume, we can write:

$$dE=TdS-pdV \tag{2}$$

If we identify the evolution of the Universe with an isentropic process, from the previous relation we obtain:

$$dE+pdV=0 \tag{3}$$

According to *Mass-Energy Equivalence* [2], we have:

$$E=Mc^2 \tag{4}$$

Obviously, we can write:

$$\frac{dM}{dt}=\frac{d}{dt}(\rho V)=V\frac{d\rho}{dt}+\rho\frac{dV}{dt} \tag{5}$$

Taking into account *(4)* and *(5)*, from *(3)* we obtain:

$$c^2V\frac{d\rho}{dt}+c^2\rho\frac{dV}{dt}+p\frac{dV}{dt}=0 \tag{6}$$

$$V\frac{d\rho}{dt}+\left(\rho+\frac{p}{c^2}\right)\frac{dV}{dt}=0 \tag{7}$$

Since V is regarded as directly proportional to R^3, we have:

$$\frac{d\rho}{dt}=-\frac{1}{V}\frac{dV}{dt}\left(\rho+\frac{p}{c^2}\right)=-\frac{3}{R}\frac{dR}{dt}\left(\rho+\frac{p}{c^2}\right) \tag{8}$$

$$\dot{\rho}=-3\frac{\dot{R}}{R}\left(\rho+\frac{p}{c^2}\right) \tag{9}$$

The foregoing represents the so-called *Fluid Equation*.

According to Zeldovich [3], the relation between pressure and density can be expressed as follows:

$$p=(\nu-1)\rho c^2 \tag{10}$$

The value of ν, hypothesized as being constant, depends on the ideal fluid with which we identify the Universe.

From *(8)*, taking into account *(10)*, we obtain:

$$\frac{d\rho}{\rho}=-3\nu\frac{dR}{R} \tag{11}$$

As a consequence, if we denote with C the constant of integration, we can easily deduce the following:

$$\rho R^{3\nu}=C \tag{12}$$

Equation *(1)* can be evidently rewritten as follows:

$$\left(\frac{dR}{dt}\right)^2=\frac{8\pi G\rho R^{3\nu}}{3}R^{2-3\nu}+\frac{1}{3}\lambda c^2R^2-kc^2 \tag{13}$$

We can now define the underlying new constant:

$$C_\nu=\frac{8\pi G\rho R^{3\nu}}{3}=\frac{8\pi GC}{3} \tag{14}$$

By substituting the previous identity into *(13)*, we obtain:

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$$\dot{R}^2 = C_\nu R^{2-3\nu} + \frac{1}{3}\lambda c^2 R^2 - kc^2 \tag{15}$$

If we denote with ω the pulsation of the Universe we want to describe, we can carry out the following position involving the cosmological constant:

$$\lambda = -3\left(\frac{\omega}{c}\right)^2 \tag{16}$$

If we set the curvature parameter equal to zero, by substituting *(16)* in *(15)* we finally obtain:

$$\dot{R}^2 = C_\nu R^{2-3\nu} - \omega^2 R^2 \tag{17}$$

From the previous equation, we can deduce as follows:

$$\frac{dR}{dt} = \sqrt{C_\nu} R^{1-\frac{3}{2}\nu} \sqrt{1-\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)^2} \tag{18}$$

$$\frac{1}{\sqrt{C_\nu} R^{1-\frac{3}{2}\nu}} \frac{dR}{\sqrt{1-\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)^2}} = dt \tag{19}$$

$$\frac{2}{3\nu\omega} \frac{d\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)}{\sqrt{1-\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right)^2}} = dt \tag{20}$$

If we impose that the radius of curvature assumes a null value when $t=0$, from the prior equation we can deduce:

$$\sin^{-1}\left(\frac{\omega R^{\frac{3}{2}\nu}}{\sqrt{C_\nu}}\right) = \frac{3}{2}\nu\omega t \tag{21}$$

$$R^{3\nu} = \frac{C_\nu}{\omega^2}\sin^2\left(\frac{3}{2}\nu\omega t\right) = \frac{C_\nu}{2\omega^2}[1-\cos(3\nu\omega t)] \tag{22}$$

$$R = \left(\frac{C_\nu}{2\omega^2}\right)^{\frac{1}{3\nu}} [1-\cos(3\nu\omega t)]^{\frac{1}{3\nu}} \tag{23}$$

According to *(23)*, we have formally achieved a model of Universe belonging to the oscillatory class ("O Type" in Harrison's Classification) [4].

From *(14)* and *(22)*, we immediately obtain:

$$\rho = \frac{3}{8\pi G}\frac{C_\nu}{R^{3\nu}} = \frac{3\omega^2}{4\pi G}\frac{1}{1-\cos(3\nu\omega t)} \tag{24}$$

Finally, by taking into account *(16)*, we can write the foregoing equation as follows:

$$\rho = -\frac{\lambda c^2}{4\pi G}\frac{1}{1-\cos(3\nu\omega t)} \tag{25}$$

2. A Simple-Harmonically Oscillating Universe

If we set ν equal to *1/3*, from *(23)* we obtain:

$$R = \frac{C_{1/3}}{2\omega^2}[1-\cos(\omega t)] \tag{26}$$

In other terms, we have found a simple-harmonically oscillating universe characterized by a variable density whose value, taking into account *(25)*, is provided by the following relation:

$$\rho = -\frac{\lambda c^2}{4\pi G}\frac{1}{1-\cos(\omega t)} \tag{27}$$

If we denote with *A* the amplitude of the motion, taking into account *(26)*, we can immediately write:

$$A = \frac{C_{1/3}}{2\omega^2} \tag{28}$$

Finally, denoting with R_m the mean radius, we have:

$$\omega t = \frac{\pi}{2} \to R = A = R_m \tag{29}$$

$$R = R_m[1-\cos(\omega t)] \tag{30}$$

$$\omega t = \frac{\pi}{2} \to \rho_m = \rho(R_m) = -\frac{\lambda c^2}{4\pi G} \tag{31}$$

From *(12)*, since ν has been set equal to *1/3*, we have:

$$\rho R = \rho_m R_m \tag{32}$$

From *(14)*, *(28)*, *(30)* and *(32)*, we have:

$$\omega^2 = \frac{C_{1/3}}{2R_m} = \frac{4\pi G}{3}\rho\frac{R}{R_m} = \frac{4\pi G\rho_m}{3} \tag{33}$$

$$(\omega R_m)^2 = \frac{2\left(\frac{2}{3}\pi R_m^3\rho_m\right)G}{R_m} \tag{34}$$

We can now carry out the following noteworthy positions:

$$M_m = \frac{2}{3}\pi R_m^3\rho_m \tag{35}$$

$$\omega R_m = c \tag{36}$$

The position in *(35)*, at a first glance undoubtedly puzzling, will be at a later time easily understood when dealing with the concept of "global symmetry".

From *(34)*, taking into account *(35)* and *(36)*, denoting with R_s the so-called Schwarzschild radius [5] [6], we obtain:

$$R_m = \frac{2M_mG}{c^2} = R_s(M_m) \tag{37}$$

In the light of the outcomes so far achieved, we can write the following relations:

$$\omega t = \frac{ct}{R_m} = \alpha \tag{38}$$

$$R = R_m(1-\cos\alpha) \tag{39}$$

$$\cos\alpha = 1-\frac{R}{R_m} \tag{40}$$

$$\dot{R} = \frac{dR}{dt} = c\sin\alpha \tag{41}$$

$$\ddot{R} = \frac{d\dot{R}}{dt} = c\omega\cos\alpha = \frac{c^2}{R_m}\left(1-\frac{R}{R_m}\right) \tag{42}$$

The beginning of a new cycle *(t=0)* occurs when the radius of curvature assumes a null value. The problem related to the singularity at *R=0*, herein not addressed, may be solved by postulating a quantized space: in other terms, we should impose some sort of quantum "bounce" (actually, the concept is anything but a novelty) [7] [8] [9] so as to prevent the radius from concretely assuming a null value.

The evolution of the hypothesized Universe is evidently characterized by four consecutive phases: an accelerated

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expansion, a decelerated expansion, a decelerated contraction, an accelerated contraction. All the above-mentioned phases have the same duration.

By taking into account *(39)* and *(41)*, we can immediately write the Hubble parameter [10], commonly denoted by *H*, as follows:

$$H = \frac{\dot{R}}{R} = \frac{c}{R_m}\frac{2\sin\left(\frac{\alpha}{2}\right)\cos\left(\frac{\alpha}{2}\right)}{2\sin^2\left(\frac{\alpha}{2}\right)} = \frac{c}{R_m}\frac{1}{\tan\left(\frac{ct}{2R_m}\right)} \quad (43)$$

As a consequence, it is quite evident how the Hubble parameter may have assumed in the past, and could possibly assume in the future, negative values.

II. A DIFFERENT POINT OF VIEW

1. Mass - Energy Equivalence: Alternative Deduction

Let's consider a material point whose motion is defined by equation *(39)* (in other terms, a simple harmonic oscillator consisting of a mass and an ideal spring). If we denote with *m* the mass of the above-mentioned point, the elastic constant, denoted by *K*, can be written as follows:

$$K = m\omega^2 = m\left(\frac{c}{R_m}\right)^2 \quad (44)$$

Consequently, the total (mechanical) energy, with obvious meaning of the notation, acquires the following form:

$$E_{R_m-point} = \frac{1}{2}KR_m^2 = \frac{1}{2}mc^2 \quad (45)$$

Now, by solely modifying the amplitude of the motion, denoted by z_m, and by keeping the values of mass and pulsation constant, we obtain:

$$z = z_m(1-\cos\alpha) \qquad z_m \in [0, R_m] \quad (46)$$

Once fixed the value of z_m, from *(39)* and *(46)* we have:

$$\frac{z_m}{R_m} = \frac{z}{R} \quad (47)$$

At any given time, the value of *R* is obviously univocally determined by means of *(39)*, being R_m a constant. On the contrary, the value of *z*, provided by *(46)*, depends on the amplitude of the motion, denoted by z_m, that can vary between zero and R_m.

The total (mechanical) energy of a material point, whose motion is defined by *(46)*, acquires the following form:

$$E_{z_m-point} = \frac{1}{2}Kz_m^2 = \frac{1}{2}\left(\frac{z_m}{R_m}\right)^2 mc^2 = \frac{1}{2}\left(\frac{z}{R}\right)^2 mc^2 \quad (48)$$

The material point can be replaced by a material segment (in other terms, it is as if we consider a spring, no longer ideal, whose length at rest is equal to R_m). The length (*R*) of the segment evolves in accordance to *(39)*.

If we denote with *M* the (constant) mass of the segment, the linear density can be defined as follows:

$$\bar{M} = \frac{M}{R} \quad (49)$$

Consequently, denoting with M_z the mass of a portion of segment characterized, at any given time, by a length equal to *z*, we can write the following:

$$M_z = z\bar{M} = \frac{z}{R}M \quad (50)$$

$$\bar{M} = \frac{M}{R} = \frac{M_z}{z} \quad (51)$$

Taking into account *(48)* and *(50)*, the energy related to an infinitesimal material segment can be written as follows:

$$dE_z = \frac{1}{2}\left(\frac{z}{R}\right)^2 c^2 dM_z = \frac{1}{2}\left(\frac{z}{R}\right)^2 c^2\bar{M}dz = \frac{Mc^2}{2R^3}z^2dz \quad (52)$$

Taking now into account *(50)* and *(52)*, the final expression for the energy of a material segment, whose length, at any given time, is equal to *z*, acquires the underlying form:

$$E_z = \int_0^z dE_z = \frac{1}{6}\left(\frac{z}{R}\right)^3 Mc^2 = \frac{1}{6}\left(\frac{z}{R}\right)^2 M_z c^2 \quad (53)$$

At this stage, in order to follow our line of reasoning, it is necessary to introduce a further spatial dimension.

The Universe we hypothesize is identifiable with a *4-ball* whose radius, denoted by *R*, evolves in accordance to *(39)*. The corresponding boundary, that represents the space we are allowed to perceive [11], is a three-dimensional surface (a *hyper sphere*) described by the following identity:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = R^2 \quad (54)$$

The *4-ball* is banally described by the following inequality:

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 \le R^2 \quad (55)$$

Let's consider the point P^+ defined as follows:

$$P^+ = (0,0,0,R) \quad (56)$$

If we denote with the P^- the antipode of P^+ (the point diametrically opposite), we have:

$$P^- = (0,0,0,-R) \quad (57)$$

We must now consider the straight line segment bordered by the points P^+ and P^- just defined.

Figures 1, 2 and *3* provide the representations of the above-mentioned segment, by looking into the scenarios that arise from *(55)* if we set equal to zero, one at a time, all the four coordinates.

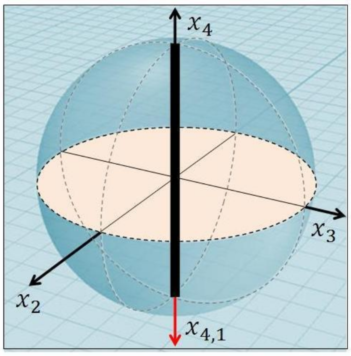

Figure 1. First Scenario (x_1=0)

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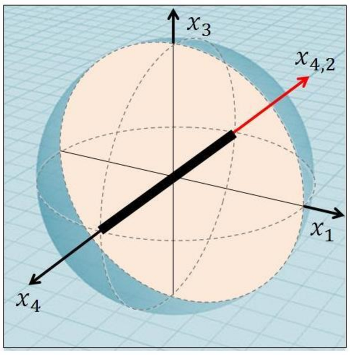

Figure 2. Second Scenario (x_2=0)

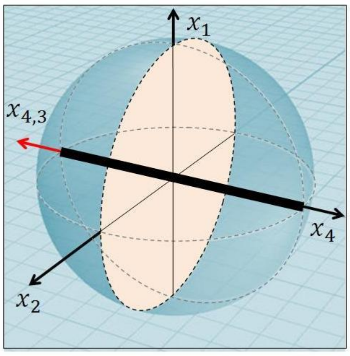

Figure 3. Third Scenario (x_3=0)

If we set $x_4=0$, we evidently obtain nothing but a single point (as shown in *Figure 4*).

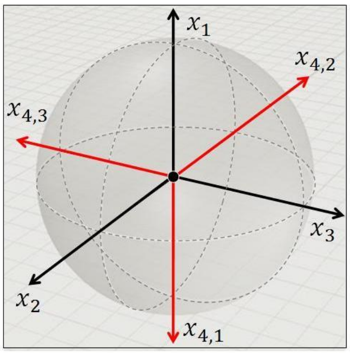

Figure 4. Fourth Scenario (x_4=0)

Therefore, we have to examine the three-dimensional scenarios that arise from the underlying identity:

$$x_i = 0 \qquad i = 1,2,3 \tag{58}$$

For example, we can set $x_1=0$ (obviously, the same line of reasoning can be followed by setting $x_2=0$ and $x_3=0$). As a consequence, from *(54)*, *(55)* and *(56)*, we immediately obtain the following:

$$x_2^2 + x_3^2 + x_4^2 \leq R^2 \tag{59}$$

$$P_1^+ = (0,0,R) \tag{60}$$

$$P_1^- = (0,0,-R) \tag{61}$$

Let's now consider the straight line segment bordered by the centre of the ball and the point defined by *(60)*.

If the segment in question, whose length evolves in accordance with *(39)*, is provided with a mass equal to *M*, its energy can be immediately deduced from *(53)* by setting *z=R*. Consequently, underlining how the same procedure can be obviously adopted for the point defined by *(61)*, we can write, with obvious meaning of the notation, as follows:

$$E_{R,1}^+ = E_{R,1}^- = \frac{1}{6}Mc^2 \tag{62}$$

Generalizing the outcome just obtained, we can write:

$$E_{R,i}^+ = E_{R,i}^- = \frac{1}{6}Mc^2 \qquad i = 1,2,3 \tag{63}$$

Consequently, continuing with the generalization, for the material segment characterized by a length equal to *2R* and a mass equal to *2M*, we have:

$$E_i = E_{R,i} = E_{R,i}^+ + E_{R,i}^- = \frac{1}{3}Mc^2 \qquad i = 1,2,3 \tag{64}$$

Finally, by superposition, we can easily write the total amount of energy related to the material segment bordered by the points defined by *(56)* and *(57)* as follows:

$$E = \sum_{i=1}^{3} E_i = Mc^2 \tag{65}$$

The points defined by *(56)* and *(57)* are nothing but the interceptions between the material segment, whose energy is provided by *(65)*, and the hyper surface described by *(54)*, that represents the Universe we are allowed to perceive when we are at rest [11] [12]. As far as our perception of reality is concerned, each point and its antipode are to be actually considered as being the same thing, since they both belong to the same straight line segment [11] [12]. In other terms, we could state that, according to our model of Universe, everything is doubled. On this subject, it is fundamental to underline how we could carry out a banal translation of the frame of reference, by setting the origin in correspondence of one of the endpoints of the material segment.

In the light of what just declared, taking into account the symmetry, the scenarios that arise from *(58)* may be alternatively represented as shown in *Figures 5*, *6* and *7*.

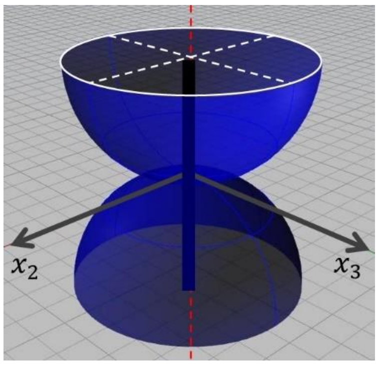

Figure 5. Alternative First Scenario (x_1=0)

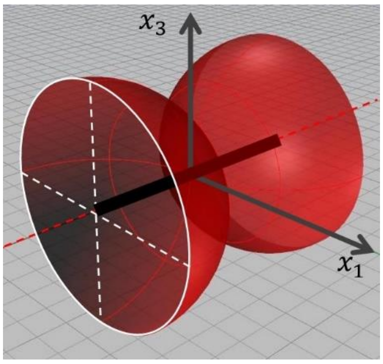

Figure 6. Alternative Second Scenario (x_2=0)

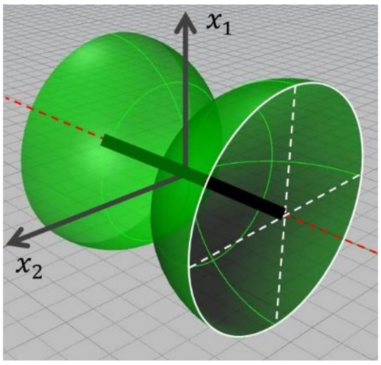

Figure 7. Alternative Third Scenario (x_3=0)

Although the topic, for the sake of brevity, is not herein addressed, it is worth highlighting how the energy in *(65)* consists of a kinetic component and a potential (elastic) component: the latter, in a certain sense, may be related to the so-called "dark energy".

2. The Conservation of Energy

Let's suppose that a material segment, characterized by an initial length equal to *2R*, starts rotating around its centre (that, by virtue of the alleged symmetry, coincides with the one of the *4-ball* with which we identify the Universe). If the total energy has to be preserved, both the length of the segment and its mass must undergo a reduction: otherwise, the kinetic energy due to the hypothesized motion should be necessarily added to the energy at rest, defined by *(65)*. Denoting with v the tangential speed of the endpoints, with $2z$ the reduced length of the segment (in motion), with $2M_z$ the corresponding (reduced) mass, and with I the moment of inertia, we can write the kinetic energy as follows:

$$E_{kinetic\ z,i} = \frac{1}{2} I \left(\frac{v}{z}\right)^2 \tag{66}$$

The moment of inertia of the segment is banally provided by the following relation:

$$I = \frac{1}{12}(2M_z)(2z)^2 = \frac{2}{3}M_z z^2 \tag{67}$$

From *(66)* and *(67)* we immediately obtain:

$$E_{kinetic\ z,i} = \frac{1}{3}M_z v^2 \tag{68}$$

The value of M_z is provided by *(50)*: in other terms, according to *(51)*, the linear density is considered as being constant (it does not vary along the radial direction).

From *(53)*, taking into account the symmetry, we can state that the above-mentioned segment, since it is involved in the cyclic evolution described by *(46)*, is also provided with an energy that, for each of the scenarios that arise from Equation *(58)*, can be written as follows:

$$E_{z,i} = E^+_{z,i} = E^-_{z,i} = \frac{1}{3}\left(\frac{z}{R}\right)^2 M_z c^2 \tag{69}$$

According to our theory [11], taking into account *(64)*, *(68)* and *(69)*, we may express the *Conservation of Energy*, for the considered scenario, as follows:

$$E_i = \frac{1}{3}Mc^2 = \frac{1}{3}M_z v^2 + \frac{1}{3}\left(\frac{z}{R}\right)^2 M_z c^2 + \frac{1}{3}(M - M_z)c^2 \tag{70}$$

By multiplying by three all the members of *(70)*, taking into account *(65)*, we easily obtain the underlying relation:

$$E = Mc^2 = M_z v^2 + \left(\frac{z}{R}\right)^2 M_z c^2 + (M - M_z)c^2 \tag{71}$$

As far as the last member of *(71)* is concerned, we may state that the first term represents the (real) kinetic energy, the second term the potential energy (related to the cyclic evolution of the Universe), while the third term (the "non-material" component, that may be related to the so-called "quantum potential") [13] [14], represents the energy needed to obtain the motion (to obtain the mass reduction).

From the previous equation we immediately deduce the underlying noteworthy identity:

$$M_z c^2 = M_z v^2 + \left(\frac{z}{R}\right)^2 M_z c^2 \tag{72}$$

According to the definition of Lorentz factor [15], we have:

$$\gamma = \frac{1}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \tag{73}$$

$$\left(\frac{v}{c}\right)^2 = \beta^2 = 1 - \frac{1}{\gamma^2} \tag{74}$$

From *(72)*, exploiting *(73)* and *(74)*, we evidently obtain:

$$z = R\sqrt{1 - \left(\frac{v}{c}\right)^2} = R\sqrt{1 - \beta^2} = \frac{R}{\gamma} \tag{75}$$

By virtue of *(50)*, we can evidently write:

$$\frac{M}{z} = \frac{R}{z}\bar{M} \tag{76}$$

Consequently, taking into account *(50)*, *(51)* and *(76)*, the specific energies (the energies per unit of length) defined in *(71)* can now be written, with obvious meaning of the notation, as follows:

$$\bar{E} = \frac{Mc^2}{z} = \frac{\bar{M}}{\frac{z}{R}}c^2 = \frac{\bar{M}c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \gamma \bar{M} c^2 \tag{77}$$

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$$\bar{E}' = \frac{M_z}{z}v^2 = \bar{M}\beta^2c^2 = \left(1 - \frac{1}{\gamma^2}\right)\bar{M}c^2 \qquad (78)$$

$$\bar{E}'' = \left(\frac{z}{R}\right)^2\frac{M_z}{z}c^2 = \frac{\bar{M}c^2}{\gamma^2} \qquad (79)$$

$$\bar{E}''' = \left(\frac{M}{M_z} - 1\right)\frac{M_z}{z}c^2 = \left(\frac{R}{z} - 1\right)\frac{M_z}{z}c^2 = (\gamma - 1)\bar{M}c^2 \qquad (80)$$

Very evidently, by virtue of the last four equations, we can concisely write *(71)* as follows:

$$E = Mc^2 = E' + E'' + E''' \qquad (81)$$

Alternatively, taking into account *(77)*, *(78)*, *(79)* and *(80)*, we can resort to the underlying extend writing:

$$\gamma\bar{M}c^2 = \left(1 - \frac{1}{\gamma^2}\right)\bar{M}c^2 + \frac{\bar{M}c^2}{\gamma^2} + (\gamma - 1)\bar{M}c^2 \qquad (82)$$

Denoting with E_0 the energy at rest, we can write:

$$\bar{E}_0 = \frac{Mc^2}{R} = \bar{M}c^2 \qquad (83)$$

$$\bar{E} = \gamma\bar{M}c^2 = \bar{E}_0 + (\gamma - 1)\bar{M}c^2 = \bar{E}_0 + \bar{E}''' \qquad (84)$$

By dividing both members of *(72)* by *z*, taking into account *(51)* and resorting to the Lorentz factor, we obtain:

$$\bar{M}c^2 = \bar{M}v^2 + \frac{\bar{M}c^2}{\gamma^2} \qquad (85)$$

By multiplying both members of the foregoing equation by the Lorentz factor, we have:

$$\gamma\bar{M}c^2 = \gamma\bar{M}v^2 + \frac{\bar{M}c^2}{\gamma} \qquad (86)$$

$$\bar{E} = \frac{\bar{M}c^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{\bar{M}v^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} + \sqrt{1 - \left(\frac{v}{c}\right)^2}\,\bar{M}c^2 \qquad (87)$$

3. The "Relativistic" Energy

In order to obtain the formal definition of the so-called *Relativistic Energy*, we have to recall the concept of "dimensional thickness", elsewhere introduced [11].

Very briefly, according to our theory, the three-dimensional curved space we are allowed to perceive may be characterized by a thickness, denoted by Δz_{min}, that may represent nothing but the (radial) "quantum of space".

Consequently, the mass we perceive, denoted by *m*, may be provided by the underlying banal relation:

$$m = \bar{M}\Delta z_{min} \qquad (88)$$

As for the energy we perceive, with obvious meaning of the notation, we can write:

$$E_m = \bar{E}\Delta z_{min} = (\bar{E}' + \bar{E}'' + \bar{E}''')\Delta z_{min} \qquad (89)$$

$$E_m = E'_m + E''_m + E'''_m \qquad (90)$$

By multiplying both members of *(82)* by Δz_{min}, we have:

$$E_m = \gamma mc^2 = \left(1 - \frac{1}{\gamma^2}\right)mc^2 + \frac{mc^2}{\gamma^2} + (\gamma - 1)mc^2 \qquad (91)$$

Finally, by multiplying all the members of *(87)* by Δz_{min}, we obtain the well-known underlying equation:

$$E_m = \frac{mc^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} = \frac{mv^2}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} + \sqrt{1 - \left(\frac{v}{c}\right)^2}\,mc^2 \qquad (92)$$

Denoting with *p* the momentum, with *L* the (relativistic) Lagrangian, and with *H* the Hamiltonian, we have:

$$p = \frac{mv}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \qquad (93)$$

$$L = -\sqrt{1 - \left(\frac{v}{c}\right)^2}\,mc^2 \qquad (94)$$

$$E_m = H = pv - L \qquad (95)$$

Let's now define the angular speed as follows:

$$\dot{\chi} = \frac{d\chi}{dt} = \frac{v}{z} \qquad (96)$$

According to *(71)*, As a consequence, from the point of view of an observer at rest, the value of the (tangential) speed of the endpoints of the rotating segment is greater than *v*. Therefore, taking into account *(75)* and *(96)*, we can define the perceived (virtual) speed [12], denoted by *v**, as follows:

$$v^* = \dot{\chi}R = \frac{v}{\sqrt{1 - \left(\frac{v}{c}\right)^2}} \qquad (97)$$

It is worth underlining how, according to *(71)*, the rotating segment (perceived as a translating point) may also exhibit a wave-like behavior. In particular, there is no mass in the range *]z, R]*. Consequently, taking into account *(75)* and *(97)*, denoting with *h* the Planck constant, we can immediately write the corresponding de Broglie (relativistic) length [16] as follows:

$$\lambda_R = \frac{h}{mv^*} = \frac{h}{mv}\sqrt{1 - \left(\frac{v}{c}\right)^2} \qquad (98)$$

4. Friedmann – Lemaître Equations: alternative deduction

Our analysis will be carried out by considering one amongst the scenarios that arise from *(54)* and *(55)*, once having set equal to zero one of the coordinates. In other terms, the Universe in its entirety is identified with a ball, and the curved space we are allowed to perceive is assimilated to a spherical surface.

Taking into account the "global symmetry" so far hypothesized, denoting with *M* half the mass of the Universe, we can define the density as follows:

$$\rho = \frac{M}{\frac{2}{3}\pi R^3} \qquad (99)$$

If M_m represents the mass when $R=R_m$, from the prior equation, coherently with *(35)*, we obtain:

$$\rho_m = \rho(R_m) = \frac{M_m}{\frac{2}{3}\pi R_m^3} = \frac{3\frac{c^2}{R_m^2}}{4\pi\frac{c^2 R_m}{2M_m}} \qquad (100)$$

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We can now carry out, coherently with *(37)*, the following noteworthy position [17]:

$$G = \frac{R_m c^2}{2M_m} \tag{101}$$

From the previous identity, by virtue of which we may identify R_m with the Schwarzschild radius of the Universe [5] [6] [17], taking into account *(100)*, we obtain:

$$\rho_m = \frac{3\frac{c^2}{R_m^2}}{4\pi G} \tag{102}$$

We can now define the cosmological constant as follows:

$$\lambda = -\frac{3}{R_m^2} \tag{103}$$

In accordance with the foregoing position, from *(102)* we immediately obtain what already deduced in *(31)*.

If we identify the evolution of the Universe with an isentropic process, taking into account the relation between pressure and density, denoting with V_m the mean volume, we can write:

$$\frac{d}{dt}(pV^\nu) = 0 \tag{104}$$

$$\frac{d}{dt}(\rho V^\nu) = 0 \tag{105}$$

$$\rho V^\nu = \rho_m V_m^\nu \tag{106}$$

$$\rho R^{3\nu} = \rho_m R_m^{3\nu} \tag{107}$$

The Newtonian gravitational field produced by a generic mass *m* can be written as follows:

$$g = \frac{Gm}{d^2} \tag{108}$$

We can approximatively identify *d* with the measured distance between the gravitational source and the point in correspondence of which we want to evaluate the field. Alternatively, taking into account a possible pseudo-Newtonian gravity, whose expression should obviously resemble *(108)*, we could simply impose a linear dependence between *d*, that would no longer be identifiable with the measured distance, and the radius of the Universe. Hence, for a generic source *m*, once fixed the angular distance (as perceived by an observer placed at the center of the ball with which we identify the Universe), we may write, with obvious meaning of the notation, the following:

$$d = d(R, \chi) \propto R \tag{109}$$

$$g \propto \frac{m}{R^2} \tag{110}$$

To maintain the field constant, generalizing *(110)*, we must necessarily write [17]:

$$\frac{d}{dt}\left(\frac{M}{R^2}\right) = 0 \tag{111}$$

$$\frac{M}{R^2} = \frac{M_m}{R_m^2} \tag{112}$$

From the previous, we obtain what already deduced in *(32)*. By comparing *(32)* to *(107)*, we have:

$$\nu = \frac{1}{3} \tag{113}$$

The previous is the value of ν we have resorted to in order to achieve a Simple-Harmonically Oscillating Universe starting from the Friedmann – Lemaître Equations.

Evidently, we consider the variations of cosmological distances as being exclusively "metric": in other words, we are postulating that the amount of space between whatever couple of points remains the same with the passing of time (on this subject, it could be worth bearing in mind how Hubble himself started bringing into question the relation between the redshift and the recessional velocity of astronomical objects) [18]. As a consequence, if we assign a variable value to cosmological distances, coherently with the apparent evolution of the Universe, we also have to assign, to maintain the gravitational field constant, a variable value to the mass that produces the field itself. On this subject, although the topic, for the sake of brevity, is not herein addressed, we hypothesize that the so-called cosmological redshift may be a phenomenon banally related to the conservation of energy. As well known, the energy of a quantum of light can be expressed as the product between the value of its frequency and the Plank constant. On the one hand, as an alternative to the conventional interpretation of the cosmological redshift, we could accept that, in travelling through the interstellar vacuum, light may somehow "get tired", so as losing part of its energy [19] [20] [21]. On the other hand, we may simply imagine that the Plank constant could vary over time [22] [23]: consequently, in order to preserve its energy, a photon could be forced into modifying its frequency (and its length).

Now, by taking into account *(10)*, we can write:

$$p = -\frac{2}{3}\rho c^2 \tag{114}$$

From *(32)* and *(102)* we immediately deduce the following:

$$\rho = \frac{R_m}{R}\rho_m = \frac{3}{4\pi G}\frac{c^2}{RR_m} \tag{115}$$

$$\frac{c^2}{RR_m} = \frac{4\pi G}{3}\rho \tag{116}$$

From *(40)* and *(41)* we easily obtain:

$$\dot{R}^2 = c^2(1-\cos^2\alpha) = 2c^2\frac{R}{R_m} - c^2\frac{R^2}{R_m^2} \tag{117}$$

$$\dot{R}^2 + c^2\frac{R^2}{R_m^2} = 2c^2\frac{R}{R_m} \tag{118}$$

If the radius is different from zero, considering the previous relation, by virtue of *(116)*, we have:

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{c^2}{R_m^2} = 2\frac{c^2}{RR_m} = \frac{8\pi G}{3}\rho \tag{119}$$

Taking into account *(102)*, from the foregoing we obtain:

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{\lambda c^2}{3} = \frac{8\pi G}{3}\rho \tag{120}$$

$$\left(\frac{dR}{dt}\right)^2 = \frac{1}{3}(8\pi G\rho + \lambda c^2)R^2 \tag{121}$$

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Obviously, the previous equation is nothing but *(1)* with the curvature parameter equal to zero.

Now, we may easily rearrange *(117)* as follows:

$$\dot{R}^2 = 2R\frac{c^2}{R_m}\left(1 - \frac{R}{R_m}\right) + c^2\frac{R^2}{R_m^2} \tag{122}$$

From the foregoing equation, by virtue of *(42)* and *(103)*, we can deduce:

$$\dot{R}^2 = 2R\ddot{R} + c^2\frac{R^2}{R_m^2} = 2R\ddot{R} - \frac{\lambda c^2}{3}R^2 \tag{123}$$

$$\left(\frac{\dot{R}}{R}\right)^2 = 2\frac{\ddot{R}}{R} - \frac{\lambda c^2}{3} \tag{124}$$

From *(120)*, taking into account *(114)*, we can easily deduce the following:

$$\left(\frac{\dot{R}}{R}\right)^2 - \frac{\lambda c^2}{3} = -\frac{4\pi G}{c^2}p \tag{125}$$

If we multiply by two the first and second member of the previous equation, we immediately obtain:

$$2\left(\frac{\dot{R}}{R}\right)^2 - \frac{2}{3}\lambda c^2 = \left(\frac{\dot{R}}{R}\right)^2 + \left(\frac{\dot{R}}{R}\right)^2 - \frac{2}{3}\lambda c^2 = -\frac{8\pi G}{c^2}p \tag{126}$$

From the previous, by taking into account *(124)*, we finally obtain the second Friedmann- Lemaître equation:

$$2\frac{\ddot{R}}{R} + \left(\frac{\dot{R}}{R}\right)^2 - \lambda c^2 = -\frac{8\pi G}{c^2}p \tag{127}$$

III. TOWARDS A NEW CONCEPT OF CLOSED (AND OPEN) SYSTEM: THE EM - DRIVE

1. EM-Drive: Brief Introduction

Very qualitatively, the *EM-Drive* is nothing but a resonant cavity fuelled by microwaves, basically consisting of a hollow conical frustum and a magnetron. According to Shawyer [24], the principle of operation of his revolutionary contraption is essentially based on the radiation pressure: in a few words, the alleged thrust would arise from the difference between the forces exerted upon the reflectors (the bases of the frustum). In spite of the fact that such a device, as long as it is considered as being a closed system, explicitly violates the conservation of momentum and Newton's well-known third law, it would appear, according to several tests to date, that the *EM-Drive* can concretely deliver a certain thrust without a detectable exhaust [25]. As implicitly suggested in the foregoing sentence, the easiest way to solve the paradox may consist in demonstrating, first and foremost, that the device in question cannot be properly regarded as a closed system.

For the sake of clarity, we reveal in advance that the detectability of the exhaust [26], a term that actually will turn out to be not entirely suitable for the hypothesized scenario, is not herein addressed.

2. EM-Drive: Reflectors Temperature

If something can be heated, it is surely characterized by a microstructure. Obviously, this intuitive concept also applies to the *EM-Drive* reflectors. Very approximately, when a solid is heated, its atoms start vibrating faster (around points that can be considered as being fixed). In other terms, as the temperature increases, the average kinetic energy increases (and vice versa). Several thermal analyses of the *EM-Drive* have shown how the bases of the above-mentioned device (when in operation) reach different temperatures [27]. For the sake of simplicity, we ignore how the temperature is distributed (in other terms, two generic points belonging to the same base are regarded as characterized by the same temperature). Consequently, let's denote with T_1 and T_2 the average temperatures reached by the bases (with T_2 greater than T_1).

The scenario is qualitatively depicted in *Figure 8*.

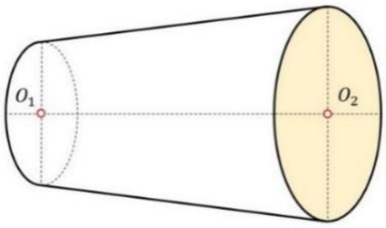

Figure 8. Hollow Conical Frustum

According to the model herein briefly expounded, O_1 and O_2, the centres of the bases, are not the endpoints of a horizontal straight line segment (ideal, since the cavity is empty). When the device is completely at rest, O_1 and O_2 can be approximately considered as being the endpoints of an (ideal) arc of circumference whose radius is equal to R. Moreover, bearing in mind the model herein exploited, the above-mentioned points are actually straight line segments whose radial extension at rest, net of the symmetry, equates the radius (of curvature) of the Universe.

3. Is the EM – Drive a Closed System?

At the beginning, when the device is not in operation, the bases are characterized by the same temperature, and the *EM-Drive* can be obviously regarded as a closed system. When the device is in operation, the bases, after a certain time, reach the temperatures T_1 and T_2. Consequently, we can (statistically) state that the average kinetic energy (and, consequently, the average vibrational speed) of the points belonging to *Surface 1* is less than the average kinetic energy of the points belonging to *Surface 2*. According to the theory we have being resorting to, this means that, net of the symmetry, the radial extension of the material segment that corresponds to O_1, denoted by z_1, is greater than the one that corresponds to O_2, denoted by z_2.

The scenario is qualitatively depicted in *Figure 9*.

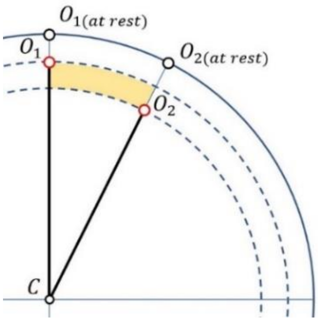

Figure 9. The "Hidden" Exhaust

In other terms, we have:

$$\overline{CO_2} = z_2 < z_1 = \overline{CO_1} \tag{128}$$

Since the electromagnetic radiation can propagate at any level [12] (for any value of z less than or equal to R), photons are allowed to leave the cavity if z is greater than z_2 (and the thrust is so legitimized). On balance, notwithstanding our perception of reality, the *EM-Drive* can be considered as being a closed system only for z less than z_2.

IV. FINAL REMARKS AND CONCLUSIONS

Firstly, it is worth highlighting how the dissertation concerning the EM – Drive has been carried out by introducing several heavy approximations and intentionally ignoring a great deal of subjects, among which stand out the detectability of the alleged exhaust and a more accurate description of the device. In particular, as far as the principle of operation of the *EM-Drive* is concerned, we have evidently avoided discussing Shawyer's explanation [24] (who, among other things, explicitly resorts to Special Relativity) [13], limiting ourselves to referring to the contents of the official *EM-Drive* page. However, the aim of this paper fundamentally lies in qualitatively providing an alternative explanation to the alleged functioning of the device, by implicitly achieving a new definition of closed system.

According to our theory, if a material point (actually a material segment) is provided with a certain kinetic energy, its radial coordinate (the radial extension of the material segment, net of the symmetry) is different from R: on this subject, we underline that if z^* is the value taken by the radial (de facto hidden) coordinate, there is no mass for z greater than z^*. Consequently, radiation (but not mass) can, as it were, pass through the point (the segment). The third addend in the second member of *(71)*, that represents the energy needed to produce the motion (in this specific case vibrational), is clearly related to the non-material component of the particle. To this extent, although the wave-particle duality is not herein addressed, we would like to underline, once again, how the above-mentioned energetic component is somehow connected to the well-known concept of quantum potential. Ultimately, we may state that the *EM-Drive* may be simultaneously regarded as being both a closed and an open system. More precisely, the device is completely closed when it is concretely at rest (actually, this is an ideal condition), and partially closed when it is in operation. Moreover, the opening of the (hidden) exhaust basically depends on the difference between the reflectors temperatures.

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